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Proper motion of γ-rays from microhalo sources

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I discuss the prospects of detecting the smallest dark matter bound structures present in the Milky Way by searching for the proper motion of γ-ray sources in the upcoming GLAST all sky map. I show that for dark matter particle candidates that couple to photons the detection of at least one γ-ray microhalo source with proper motion places a constraint on the couplings and mass of the dark matter particle. For SUSY dark matter, proper motion detection implies that the mass of the particle is less than 500 GeV and the kinetic decoupling temperature is in the range of [4-100] MeV.

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In the ΛCDM cosmological model, $\sim$ 26% of the present-day energy density of the Universe is in the form of dark matter [1] which remains an outstanding problem in cosmology and particle physics. Favored Cold dark matter (CDM) particle candidates (see [2] and references therein) are Weakly Interacting Massive Particles (WIMPs). They are kept in thermal equilibrium through interactions of the form $\chi\chi \leftrightarrow f\bar{f}$, and in kinetic equilibrium through $\chi f \leftrightarrow \chi f$. The temperature at which these particles decouple from the radiation fluid is called the freeze-out temperature, while the temperature at which they decouple from scattering interactions is called the kinetic decoupling temperature (T_d) [3, 4, 5].

In the CDM paradigm, the first collapsed and virialized dark matter structures, *microhalos*, are formed at high redshifts[6, 7, 8]. Their minimum mass is set by the RMS dark matter particle velocities set at kinetic decoupling; scales smaller than the free-streaming scale are smoothed out. For the case of supersymmetric (SUSY) cold dark matter (CDM), this mass scale is $M_{\rm m} \approx 10^{-4} M_\odot \, (T_d/10\,{\rm MeV})^{-3}$ [4, 6, 9]. Recent numerical simulations of the formation of microhalos in the context of SUSY CDM provided hints on their properties, and on their evolution and survival in the hierarchical structure formation scenario [10, 11]. Their potential detection using γ-rays has been studied in Refs. [11, 12]. Based on simple analytical studies, it has been argued that the abundance of microhalos in the Milky Way can be severely reduced due to interactions in early rapid merging processes and in stellar and galactic disc interactions [13, 14, 15, 16]. These arguments are still under debate and need to be resolved through detailed numerical simulation studies.

Investigating the existence of microhalos in the Milky Way could be very important for two reasons: 1) A possible detection can provide information about the particle physics properties of the dark matter particle, and 2) a measured abundance in the Milky Way halo contains information on the hierarchical assembly of dark matter halos at very early times, a task that is unattainable by any other method. If a fraction of microhalos survive in the present day Milky Way halo, and the dark matter particle couples to photons (as is the case for SUSY CDM as well as candidates that originate from theories with extra-dimensions), they may appear as γ-ray sources in the sky with potentially detectable proper motion [14]. A search for proper motion of unidentified γ-ray sources can provide important information on the physics of the dark matter particle and on the survival rate of microhalos in the Solar neighborhood. In this paper, I present proper motion signals that should be expected in the upcoming GLAST γ-ray experiment (http://glast.gsfc.nasa.gov) for dark matter particle candidates that couple to photons. I find that for SUSY CDM particles with mass less than $M_\chi \leq$ 500 GeV and kinetic decoupling temperature of $T_d \sim [4-100]$MeV there will be at least one detectable microhalo with proper motion if $\sim$ 20% of microhalos at the Solar radius survive [15, 16].

Photons from Microhalos: The number of photons with energy greater than $E_{\rm th}$ on Earth from a microhalo at distance $\mathcal{D}$ is

$$N_{\gamma,{\rm s}}(E > E_{\rm th}) = \frac{1}{4\pi} \frac{N_\gamma \langle \sigma v \rangle}{M_\chi^2} g[\rho(r)] \mathcal{A}_{\rm eff} \tau_{\rm exp}. \quad (1)$$

Here, $\langle \sigma v \rangle$ is the annihilation cross section of the dark matter particle to photons, M_χ is the mass of the dark matter particle and $N_\gamma = \int_{E_{\rm th}}^{M_\chi} [dN_\gamma/dE] dE$ is the total number of photons emitted above a threshold energy $E_{\rm th}$ and dN_γ/dE is the spectrum of the emitted photons. The effective area of the detector is $\mathcal{A}_{\rm eff}$, and $\tau_{\rm exp}$ is the exposure time. The term $g[\rho(r)]$ contains all the information about the distribution of dark matter in a microhalo at a distance $\mathcal{D}$, $g[\rho(r)] = \mathcal{D}^{-2} \int \rho^2(r) d^3 r = 2\pi \int_0^{\phi_m} \sin\psi \{ \int \rho^2 [r(s, \psi, \mathcal{D})] \, ds \} \, d\psi$. The integral is performed along the line of sight, for an angular extent of $\phi_m = \tan^{-1}[r_s/\mathcal{D}]$, where r_s is the scale radius. The dis-

tribution of mass is $\rho(r)$ and is assumed to be described by an NFW profile [17] (for an NFW profile, $\sim$90% of the flux comes within the region inside of r_s). All microhalos are described by the same value of the quantity $g[\rho(r)]$ if all are formed at the same redshift, $z_{\rm form} \approx 70$ [10]. Given a concentration parameter $c_{\rm v} \approx 1$ [10], a microhalo of mass $M_{\rm m}$ is described by an NFW profile with a characteristic density of $\rho_s \approx 756$ GeV cm^{-3} and a scale radius $r_s \approx 3 \times 10^{-3}\,{\rm pc} M_{\rm m,-6}^{1/3} Z_{70}^{-3}$, where $M_{\rm m,-6} = M_{\rm m}/10^{-6}\,M_\odot$ and $Z_{70} = [(1+z_{\rm form})/71]$ For the case of SUSY CDM, the number of photons detected on Earth that originate within a microhalo region defined by the PSF of GLAST can be written as

$$N_{\gamma,{\rm s}} \approx 50 \left(\frac{M_{\rm m,-6}}{\mathcal{D}_{0.05}^2}\right)\left(\frac{N_\gamma}{30}\right)\left(\frac{\langle\sigma v\rangle_{-26}}{M_{\chi,40}{}^2}\right) Z_{70}^3. \qquad (2)$$

Here, $\mathcal{D}_{0.05} = \mathcal{D}/0.05$pc, the threshold energy is $E_{\rm th} = 3$ GeV and the Hubble parameter is taken as $h = 0.7$. In this expression, $\langle\sigma v\rangle_{-26} = \langle\sigma v\rangle/10^{-26}{\rm cm}^3{\rm s}^{-1}$, $M_{\chi,46} = M_\chi/40$ GeV, and the photon spectrum is taken from Ref. [18]. This particular choice of cross section, particle mass and threshold energy represents an optimistic approach to the case of SUSY CDM detected with GLAST [19], because the annihilation cross section can be up to a factor of 10^6 smaller, while the mass can be as high as numerous TeV. However, for dark matter candidates that arise from stable Kaluza-Klein excitations in models with universal extra dimensions the annihilation cross section to photons can be much higher [2].

In order to detect the γ-ray signal from microhalos, the source signal must be larger than the threshold number of photons for detection, i.e., $N_{\gamma,{\rm s}} \geq N_{\gamma,{\rm th}}$, where $N_{\gamma,{\rm th}} \approx 6$ for a GLAST orbit-averaged effective area of $\mathcal{A}_{\rm eff} \approx 2\times10^3{\rm cm}^2$ [20] and an exposure time of 2 years. Given a microhalo of mass $M_{\rm m}$, the maximum distance from the Sun $\mathcal{D}_{\rm max}$ at which a microhalo can be detected is obtained by setting Eq. 1 equal to $N_{\gamma,{\rm th}}$. I define the "visibility volume" of a microhalo of mass $M_{\rm m}$ to be the volume defined through $\mathcal{D}_{\rm max}$.

Microhalos formed at $z_{\rm form} \approx 70$ will merge to form present day dark matter halos. A fraction of the Milky Way halo mass perhaps remains in the form of microhalos formed at very high redshifts. Simulations show that the subhalo mass function in Milky Way size halos is $dN/dM \sim M^{-2}$, with about 10% of the mass of the Milky Way in halos of mass greater than $10^7 M_\odot$. Analytical arguments based on approximations for the survival probability of microhalos in an evolving parent halo [15], as well as preliminary numerical work on the mass function of subhalos down to microhalo scales [11], find the form of the subhalo mass function is preserved down to microhalo scales. Thus, the amount of mass per logarithmic mass interval is a constant, and depends only on the survival probability of microhalos. If approximately 20% of microhalos survive [15] in the Solar neighborhood (with perhaps a small mass dependence on microhalo mass [16]), then the fraction of the local mass in microhalos is roughly of order 0.2%. I define the quantity ξ to be the fraction of the dark matter density in the solar neighborhood that is in the form of microhalos, i.e., $\xi = 0.002$. If there are $N_{\rm m}$ microhalos in a logarithmic interval in $M_{\rm m}$, the number of microhalos per radial distance interval from the Sun is $dN_{\rm m}/d\mathcal{D} = 4\pi\xi\mathcal{D}^2\rho(r_\odot)/M_{\rm m}$, and the total number of microhalos per logarithmic mass interval in $M_{\rm m}$ within a visibility volume is $N_{\rm m} = \int_0^{\mathcal{D}_{\rm max}}[dN_{\rm m}/d\mathcal{D}]d\mathcal{D}$. The Sun is located at $r_\odot \approx 8$ kpc from the galactic center, and the local dark matter density is $\rho(r_\odot) = 0.01\,M_\odot\,{\rm pc}^{-3}$ [21].

Microhalo proper motion: I assume microhalos obey a Maxwell-Boltzmann velocity distribution function set by the virial temperature of the Milky Way halo. On the Earth's frame, the distribution function can be written as $f(v)dv = [v\zeta(v)/\pi^{1/2}v_e v_0]dv$. where, $\zeta(v) = \exp[(v_1/v_0)^2] - \exp[(v_2/v_0)^2]$, $v_1 = v - v_e$ and $v_2 = v + v_e$. Here, $v_0 \approx 220$ km s^{-1} is the velocity of the Sun in the frame of the Galaxy, and $v_e = 1.12v_0$ is the speed of the Earth (the very small annual modulation of the Earth's velocity is neglected). A microhalo at a distance $\mathcal{D}$ from the Sun, and with a velocity with magnitude v at an angle θ relative to the Earth's frame will exhibit a proper motion of angular displacement $\mu = \tan^{-1}[\tau v\sin\theta/\mathcal{D}]$ in a time period τ. Naively, for GLAST, a detection of proper motion requires that the angular displacement must be greater than the point spread function, i.e., $\mu > \mu_0 = 9$ arcminutes (which is the 68% PSF for GLAST). However, the PSF of GLAST improves to less than 9 arcminutes at energies greater than few GeV because the sensitivity of the instrument is not background dominated. It is reasonable to assume that a detection of proper motion is defined by the condition $\mu_{\rm th} > \mu_0/\sqrt{N_{\gamma,{\rm th}}} \approx 4\,$arcmin. This is a conservative approach, because large number of photon counts (as might be the case for a nearby microhalo) may in fact increase the angular resolution of the detector [22]. Furthermore, it is possible that some microhalos are close enough that they will have an angular extent (see Fig. 2) such that the excess count in adjacent angular bins may be used to even better localize the source, increasing the sensitivity of the detector to proper motion measurements. The number of microhalos in a logarithmic interval in $M_{\rm m}$ detected with GLAST with proper motion greater than $\mu_{\rm th}$ is obtained by integrating the velocity distribution function with the abundance of microhalos with a velocity vector at an angle greater than $\theta_{\rm min}(\mu_{\rm th}, v, \mathcal{D}) = \sin^{-1}[\mathcal{D}\tan\mu_{\rm th}/v\tau]$, as

$$\begin{aligned} N_{\rm m}(M_{\rm m}, \geq \mu_{\rm th}) &= 4\pi\int_0^{\mathcal{D}_{\rm max}(M_{\rm m})} \frac{dN_{\rm m}}{d\mathcal{D}} \qquad (3) \\ &\times \left(\int_0^\infty f(v)\cos[\theta_{\rm min}(\mu_{\rm th}, v, \mathcal{D})]dv\right) d\mathcal{D}. \end{aligned}$$

Results: Fig. 1 shows the number of visible microhalos

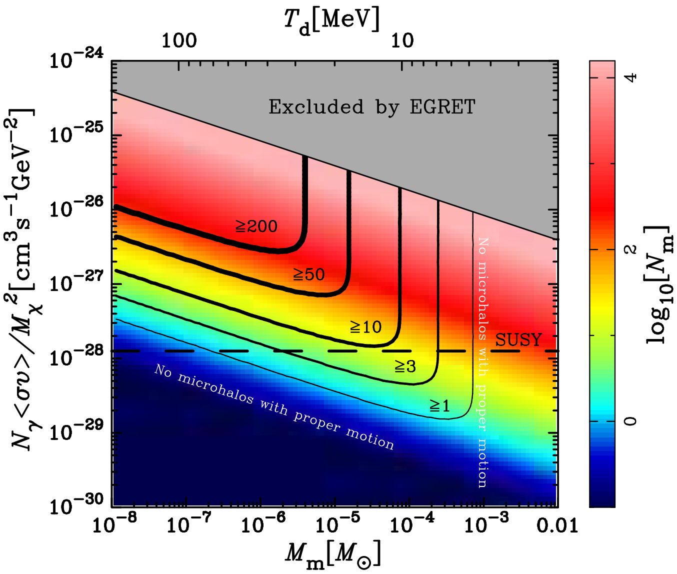

FIG. 1: *Top*: The dependence of the number of visible microhalos per logarithmic mass interval to the microhalo mass and the properties of the dark matter particle. *Solid* lines depict the iso-number contours of microhalos which also exhibit detectable proper motion. The *dashed* line shows the best case scenario for SUSY CDM particle. The shaded area shows the region that is already being excluded by EGRET measurements.

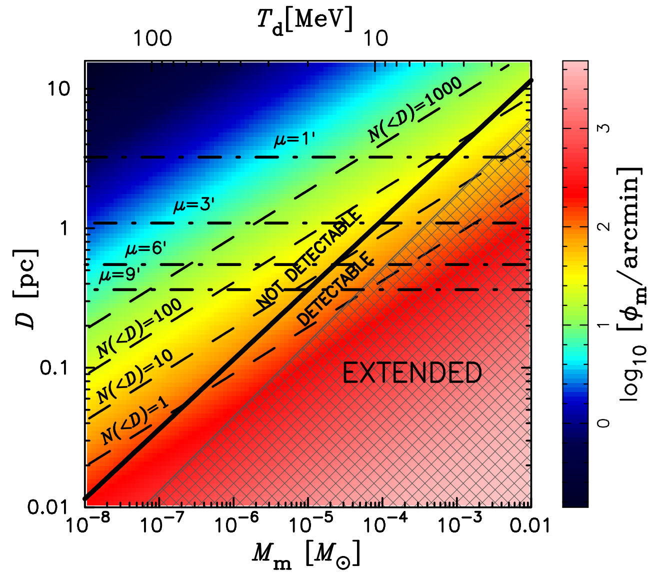

FIG. 2: The angular size of microhalos as a function of microhalo mass and distance for the best case scenario of SUSY CDM. The *solid* line defines the distance threshold for detection. *Dashed* lines depict the iso-number contours of the number of microhalos per logarithmic mass interval within a volume defined by $\mathcal{D}$. *Dot-dashed* lines show the maximum distance at which a microhalo could exhibit proper motion of 1, 3, 6 & 9 arcmin. The *cross-hatched* area corresponds to the region where microhalos have an angular extent that should be detectable by *GLAST*.

per logarithmic mass interval and contours of the number of microhalos with detectable proper motion for different microhalo masses and physics of the dark matter particle. The dashed line depicts the best case scenario for SUSY CDM, namely, a neutralino dark matter, with $M_\chi = 46$ GeV, and $\langle\sigma v\rangle = 5\times10^{-26}\mathrm{cm}^3\mathrm{s}^{-1}$. The photon spectrum is taken from Ref. [18], and is integrated from $E_{\rm th} = 4$ GeV. The integration time is 2 years. If, for example, the decoupling temperature of the SUSY CDM particle is $\sim 10\,$MeV, then microhalos have a mass of $\sim 10^{-4}\,M_\odot$, and if $\sim 0.2\%$ of them survive in the Milky Way halo to the present day, ~ 20 will be detected with GLAST, and 8 out of 20 will exhibit a detectable proper motion [27].

The shape of the contours of microhalos with detectable proper motion can be understood with the help of Fig. 2. The radius from the Sun at which a microhalo is detected is $\mathcal{D} \sim M_{\rm m}^{1/2}$. However, the radius of the volume that is needed in order to encompass at least one microhalo is $\mathcal{D} \sim M_{\rm m}^{1/3}$ (see e.g. the intersection of the $N(<\mathcal{D}) = 1$ contour and the detectability line in Fig. 2). Therefore, for small mass microhalos $M_{\rm m} \leq 10^{-6}M_\odot$, the number of microhalos with proper motion is limited by the non-abundance of microhalos within a visibility volume. On the other hand, for high mass microhalos, even though the detectable number can be high, the number of microhalos exhibiting proper motion is limited by the distance at which a microhalo can be placed and still exhibit a detectable proper motion, because $\mu \sim \mathcal{D}^{-1}$ (see e.g. the intersection of the $\mu = 3'$ and $N(<\mathcal{D}) = 1$ contour in Fig. 2).

For smaller(larger) values of ξ, such as the case where microhalos are disrupted more(less) efficiently, the number of visible microhalos and/or ones that exhibit detectable proper motion should be multiplied by a factor of $\xi/0.002$. An interesting outcome of the detection of a large number of microhalos will be the potential measurement of their parallax. Such measurements can be used to estimate the absolute luminosity and thus the measured flux can provide important information about the physics of the dark matter particle and its distribution. This calculation assumes isotropy of the microhalo distribution in the solar neighborhood and thus isotropy in the distribution of detected microhalos. However, the Doppler effect due to the motion of the Earth-Sun system in the Milky Way halo will create a dipolar distribution with an amplitude of order $\Delta N_{\rm m}/N_{\rm m} \sim 10^{-3}\%$ [23].

In the context of SUSY CDM, the detection of at least 1 microhalo with proper motion has an important implication: the decoupling temperature must be in the range $T_d \sim [4-100]$ GeV, and the mass of the particle must be $M_\chi \leq 500$ GeV. The former is inferred from the crossing of the SUSY $N_\gamma\langle\sigma v\rangle/M_\chi^2$ line with the ≥ 1 iso-number contour of detectable microhalos with proper motion in

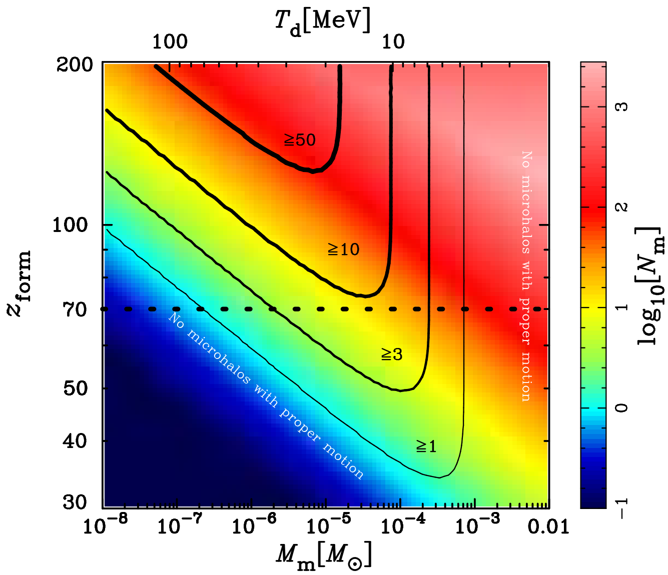

FIG. 3: The dependence of the number of visible microhalos per logarithmic mass interval on the microhalo mass and redshift of formation. *Solid* lines as in figure 1. The *dashed* line depicts the redshift at which microhalos form in numerical simulations [10, 11, 14].

Fig. 1. The latter is inferred from the lowest value of the quantity $N_\gamma \langle \sigma v \rangle / M_\chi^2$ along the same ≥ 1 iso-number contour in Fig. 1. These bounds can be understood in the following way: High decoupling temperatures imply small microhalos, which are less luminous; The visibility volume shrinks reducing the number of detectable microhalos ($N_{\rm m} \sim M_{\rm m}^{-1} \mathcal{D}^3 \sim M_{\rm m}^{1/2}$). On the other hand, low decoupling temperature results in more massive microhalos, in a larger visibility volume. Proper motion is inversely proportional to the distance of the microhalo, $\mu \sim \mathcal{D}^{-1} \sim M_{\rm m}^{-1/2}$, and so more massive microhalos will tend to exhibit smaller proper motions. It will be of interest to deduce the implications of the kinetic decoupling temperature and particle mass constraints to direct searches for dark matter, because the interactions (scalar scattering) that set the kinetic decoupling temperature are the same as the ones that are expected to take place in a direct dark matter detection experiment.

The cross-hatched region in Fig. 2 depicts the region where microhalos can potentially be resolved as extended objects (greater than the 9 arcmin PSF of GLAST). If for example microhalos have mass of order $\sim 10^{-3} M_\odot$, then there will be ~ 25 microhalos visible, out of which ~ 7 will be resolved as extended objects, and ~ 2 of them will exhibit a detectable proper motion of greater than 3 arcmin. This number is a conservative estimate since the excess photon flux from adjacent resolution bins in extended objects could be used to better localize the position of the source[28].

Fig. 3 shows the number of visible microhalos and contours of the number of microhalos with detectable proper motion for different microhalo masses and formation redshift. Recent simulations suggest that microhalos are formed at $z_{\rm form} \sim 70$ (depicted as the dashed line in Fig. 2). The assumption that microhalos form at $z_{\rm form} \sim 70$ should be taken with caution because the redshift of formation depends on 1) the rarity of the overdensity from which the microhalos collapse [8, 11] and 2) the scale invariance of the primordial power spectrum [8, 24]. Furthermore, simulations of small box-sizes are very sensitive to the redshift at which the simulation is initiated [25]. This redshift must be high enough such that the initial Zel'dovich displacement is a small enough fraction of the mean inter-particle separation. For simulations aiming at the formation of microhalos this criterion implies a starting redshift much higher than what has been assumed in refs. [10, 11, 14]. Not fullfiling this requirement delays structure formation on all scales in simulations. Therefore, if a population of microhalos is formed at a higher redshift than what has been assumed in this calculation would imply that the number of visible microhalos and microhalos with detectable proper motion could be higher as shown in Fig. 2.

In summary, a detection of the proper motion of γ-ray sources would be of profound importance. First and foremost, it will be a detection of annihilating dark matter. In addition, it will provide information on the survival rate of microhalos in the Solar neighborhood [10, 11, 13, 14, 15, 16]. Furthermore, it will provide an upper bound to the dark matter particle mass, namely, $M_\chi \leq 500$ GeV, an area in the parameter space that will be accessible for further studies at the upcoming LHC. Moreover, the kinetic decoupling temperature of the dark matter particle must be in the range $T_d = [4-100]$ MeV, dramatically restricting the current allowed range of values [26]. Therefore, it is essential that a particular analysis for the proper motion of γ-ray sources is performed by the GLAST team, and that the GLAST lifetime is maximized so as to increase the baseline for proper motion detection.

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[1] D. N. Spergel et al. (2006), astro-ph/0603449.
[2] G. Bertone et al., Phys. Rept. **405**, 279 (2005).
[3] C. Schmid et al., Phys. Rev. **D59**, 043517 (1999).

[4] S. Hofmann et al., Phys. Rev. **D64**, 083507 (2001).
[5] X. Chen et al., Phys. Rev. **D64**, 021302 (2001).
[6] V. Berezinsky et al., Phys. Rev. **D68**, 103003 (2003).
[7] A. M. Green, S. Hofmann, and D. J. Schwarz, Mon. Not. Roy. Astron. Soc. **353**, L23 (2004).
[8] A. M. Green et al., JCAP **0508**, 003 (2005).
[9] A. Loeb and M. Zaldarriaga, Phys. Rev. **D71**, 103520 (2005).
[10] J. Diemand et al., Nature. **433**, 389 (2005).
[11] J. Diemand et al. (2006), astro-ph/0603250.
[12] L. Pieri et al., Phys. Rev. Lett. **95**, 211301 (2005).
[13] H. Zhao et al. (2005), astro-ph/0508215, astro-ph/0502049.
[14] B. Moore et al. (2005), astro-ph/0502213.
[15] V. Berezinsky et al., Phys. Rev. **D73**, 063504 (2006).
[16] A. M. Green and S. P. Goodwin (2006), astro-ph/0604142.
[17] J. F. Navarro et al., Astrophys. J. **462**, 563 (1996).
[18] L. Bergstrom et al., Astropart. Phys. **9**, 137 (1998).
[19] S. Koushiappas et al., Phys. Rev. D **69**, 043501 (2004).
[20] S. Ritz (2006), private communication.
[21] J. Holmberg and C. Flynn, Mon. Not. Roy. Astron. Soc. **313**, 209 (2000).
[22] J. F. Beacom and P. Vogel, Phys. Rev. **D60**, 033007 (1999).
[23] J. Miralda-Escudé (2006), private communication.
[24] E. Bertschinger, Phys. Rev. **D74**, 063509 (2006).
[25] K. Heitmann et al., Astrophys. J. **642**, L85 (2006).
[26] S. Profumo et al. (2006), astro-ph/0603373.
[27] See EPAPS Document No.—- for a similar figure for a detector with $\mathcal{A}_{\rm eff} = 2 \times 10^8 {\rm cm}^2$ and a PSF of 6 arcmin, and for a figure which shows the cumulative number of microhalos with proper motion as a function of the cutoff mass in the microhalo mass function in the case of SUSY CDM. This document can be reached through a direct link in the online article's HTML reference section, or via the EPAPS homepage http://www.aip.org/pubservs/epaps.html
[28] See EPAPS Document No.—- for examples of the angular profile of photons emitted from extended microhalos. This document can be reached through a direct link in the online article's HTML reference section, or via the EPAPS homepage http://www.aip.org/pubservs/epaps.html